## **Technical Comments**

TECHNICAL COMMENTS are brief discussions of papers previously published in this journal. They should not exceed 1500 words (where a figure or table counts as 200 words). The author of the previous paper is invited to submit a reply for publication in the same issue as the Technical Comment. These discussions are published as quickly as possible after receipt of the manuscripts. Neither AIAA nor its Editors are responsible for the opinions expressed by the authors.

## Reply by the Authors to A. Tewari

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THE Technical Comment by A. Tewari states that some claims pertaining to the efficiency and accuracy of the formulation presented in [1] are misleading and discusses a few aspects that would support such a statement. However, when emphasizing several of such aspects, the Technical Comment is merely repeating statements that are already included in the original paper [1]. Furthermore, there are other considerations that can be important in the matter and that the Technical Comment does not contemplate. Hence, the authors of [1] believe that, although there may be different interpretations on certain issues, their claims were never misleading.

The authors of [1] would not have an argument with many of the technical aspects that are discussed in the Technical Comment. For example, the Technical Comment is absolutely correct when it states that the rational function approximation in the Laplace domain introduces only a few additional augmented states, whereas the approach of [1] will introduce a very large number of such new states. However, such a drawback is clearly emphasized in the paragraph after Eq. (117), on p. 1571 of the original paper. There is no attempt to mislead the reader, but simply to present a different formulation that, despite still having aspects that require further work, may be an approach for the solution of problems that, to this date, plague the use of rational function approximations for the representation of the unsteady aerodynamic operator for aeroelastic analysis. Moreover, Marques and Azevedo [1] give full credit to the work previously published concerning rational function approximations, including the treatment of repeated, or nearly repeated, poles addressed by Eversman and Tewari [2]. Such work has indeed improved the conditioning of the resulting eigenvalue problem, and this fact is acknowledged in the paragraphs following Eq. (40), on p. 1567 of [1].

On the other hand, the Technical Comment seems to be missing a few other aspects that are important and that weigh in favor of the formulation discussed in [1]. For instance, the Technical Comment seems to imply that the formulations, obtained using the rational

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function approximation and using the z transform, yield the same accuracy in the final aeroelastic analysis results. Calculations reported in [1] demonstrate that this is simply not true, as one can see in Fig. 8 of the original paper. Moreover, based on the experience acquired during the research there reported, Marques and Azevedo [1] indicate that the rational function approximation only allows for a (somewhat) reduced number of augmented states due to conditioning issues. In other words, the ill conditioning is also related to the number of aerodynamic states and not only to the existence of repeated poles. Therefore, although the work in [2] has solved the ill conditioning problem that comes about due to the nearly repeated poles, it has not addressed the ill conditioning due to a large number of aerodynamic states. Hence, the accuracy issues observed with the rational function approximation approach cannot simply be solved by the addition of more aerodynamic states, because the eigenvalue problem would become ill conditioned. In other words, there would be an accuracy limitation inherent to the rational function approximation, which does not happen with the z-transform approach that allows for better accuracy, at the cost of an increased number of augmented states. Finally, the fact that the z-transform formulation does not require the process of functional approximation must be considered in the effectiveness of the entire aeroelastic analysis procedure. Such a process can indeed be rather cumbersome and require additional man and computational time. In fact, the approach presented in [1] holds the promise of allowing for a full automation of the aeroelastic analysis process, which can save manpower time, although at the cost of increasing computational time due to a larger number of augmented states.

In conclusion, the authors of [1] do not believe that the statements there included were misleading. Certainly, the intention was to present a fair assessment of possible ways of handling the representation of the aerodynamic data derived from CFD solutions. Hence, although there could exist differences in opinion on a number of aspects, the authors of [1] believe that the advantages and the drawbacks of the formulation presented have been adequately addressed and emphasized in the paper. Moreover, the approach discussed in [1], that is, the concept of using digital control theory in aeroelastic analysis, is fully coherent with recent work that treats the discretized aerodynamic equations as a discrete-time system.

## References

- [1] Marques, A. N., and Azevedo, J. L. F., "A z-Transform Discrete-Time State-Space Formulation for Aeroelastic Stability Analysis," *Journal of Aircraft*, Vol. 45, No. 5, Sept.–Oct. 2008, pp. 1564–1578. doi:10.2514/1.32561
- [2] Eversman, W., and Tewari, A., "Consistent Rational-Function Approximation for Unsteady Aerodynamics," *Journal of Aircraft*, Vol. 28, No. 9, Sept. 1991, pp. 545–552. doi:10.2514/3.46062

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